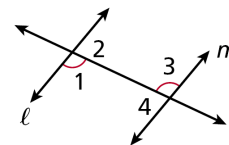


Attendance Problems. Justify each statement. (Explain why each statement is true.)

1. $\overline{QR} \cong \overline{QR}$

2. $l \parallel m$



Evaluate each expression for $x = 12$ and $y = 8.5$.

3. $2x + 7$

4. $16x - 9$

5. $8y + 5$

I can prove that a given quadrilateral is a parallelogram.

Common Core

CC.9-12.G.CO.11 Prove theorems about parallelograms.

CC.9-12.G.GPE.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems.

CC.9-12.G.MG.3 Apply geometric methods to solve design problems.

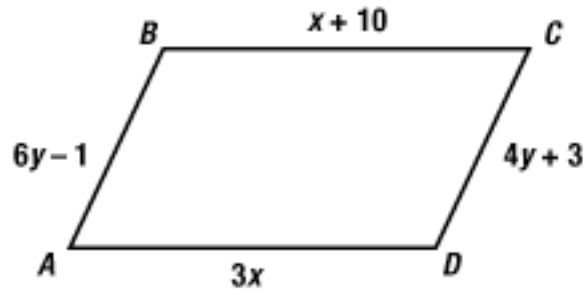
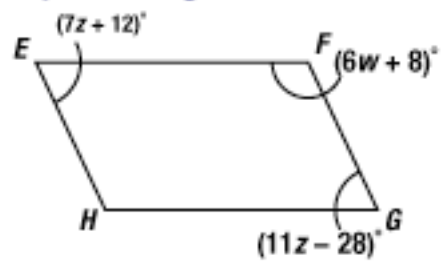
You have learned to identify the properties of a parallelogram. Now you will be given the properties of a quadrilateral and will have to tell if the quadrilateral is a parallelogram. To do this, you can use the definition of a parallelogram or the other conditions shown.

Theorems Conditions for Parallelograms

THEOREM	EXAMPLE
6-3-1 If one pair of opposite sides of a quadrilateral are parallel and congruent, then the quadrilateral is a parallelogram. (quad. with pair of opp. sides \parallel and $\cong \rightarrow \square$)	
6-3-2 If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram. (quad. with opp. sides $\cong \rightarrow \square$)	
6-3-3 If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram. (quad. with opp. $\angle \cong \rightarrow \square$)	

Theorems Conditions for Parallelograms

THEOREM	EXAMPLE
6-3-4 If an angle of a quadrilateral is supplementary to both of its consecutive angles, then the quadrilateral is a parallelogram. (quad. with \angle supp. to cons. $\angle \rightarrow \square$)	
6-3-5 If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram. (quad. with diags. bisecting each other $\rightarrow \square$)	

Video Example 1.**A.****Show that $ABCD$ is a parallelogram for $x = 5$ and $y = 2$.****B.****Show that $EFGH$ is a parallelogram for $z = 10$ and $w = 15$.**

1 Verifying Figures are Parallelograms**A** Show that $ABCD$ is a parallelogram for $x = 7$ and $y = 4$.**Step 1** Find BC and DA .

$$BC = x + 14$$

Given

$$DA = 3x$$

$$BC = 7 + 14 = 21 \quad \text{Substitute and simplify.}$$

$$DA = 3x = 3(7) = 21$$

Step 2 Find AB and CD .

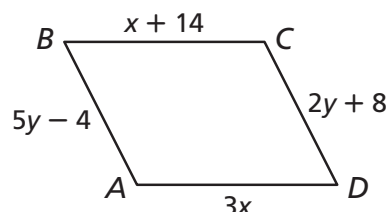
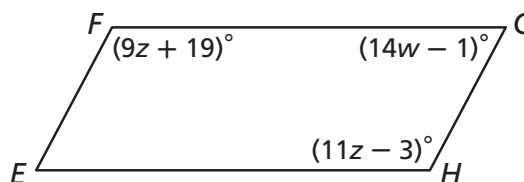
$$AB = 5y - 4$$

Given

$$CD = 2y + 8$$

$$AB = 5(4) - 4 = 16 \quad \text{Substitute and simplify.}$$

$$CD = 2(4) + 8 = 16$$

Since $BC = DA$ and $AB = CD$, $ABCD$ is a parallelogram by Theorem 6-3-2.**B** Show that $EFGH$ is a parallelogram for $z = 11$ and $w = 4.5$.

$$m\angle F = (9z + 19)^\circ$$

Given

$$m\angle F = [9(11) + 19]^\circ = 118^\circ$$

Substitute 11 for z and simplify.

$$m\angle H = (11z - 3)^\circ$$

Given

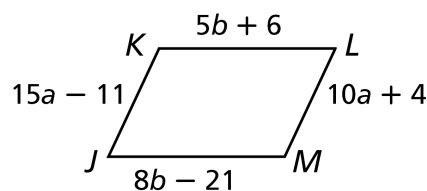
$$m\angle H = [11(11) - 3]^\circ = 118^\circ$$

Substitute 11 for z and simplify.

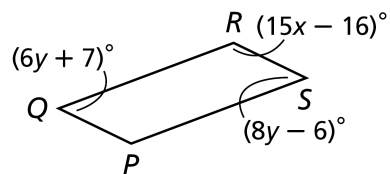
$$m\angle G = (14w - 1)^\circ$$

Given

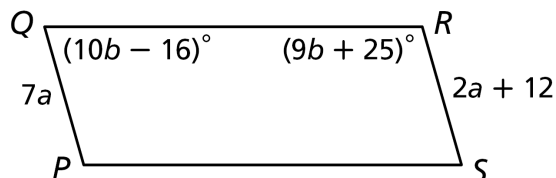
$$m\angle G = [14(4.5) - 1]^\circ = 62^\circ$$

*Substitute 4.5 for w and simplify.*Since $118^\circ + 62^\circ = 180^\circ$, $\angle G$ is supplementary to both $\angle F$ and $\angle H$. $EFGH$ is a parallelogram by Theorem 6-3-4.**Example 1.****A.** Show that $JKLM$ is a parallelogram for $a = 3$ and $b = 9$.

B. Show that $PQRS$ is a parallelogram for $x = 10$ and $y = 6.5$.

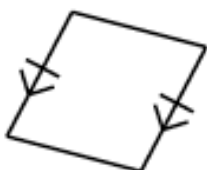


6. Guided Practice. Show that $PQRS$ is a parallelogram for $a = 2.4$ and $b = 9$.

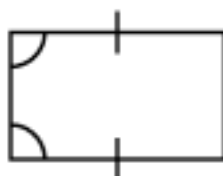


Video example 2. Determine if the quadrilateral must be a parallelogram. Justify your answer.

A.



B.

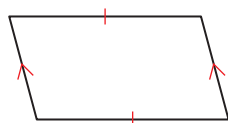


2

Applying Conditions for Parallelograms

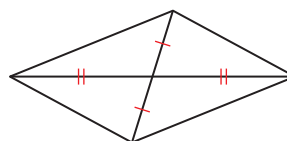
Determine if each quadrilateral must be a parallelogram. Justify your answer.

A



No. One pair of opposite sides are parallel. A different pair of opposite sides are congruent. The conditions for a parallelogram are not met.

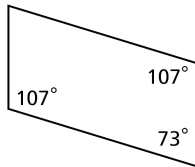
B



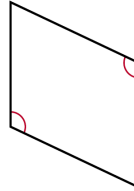
Yes. The diagonals bisect each other. By Theorem 6-3-5, the quadrilateral is a parallelogram.

Example 2. Determine if the quadrilateral must be a parallelogram. Justify your answer.

A.

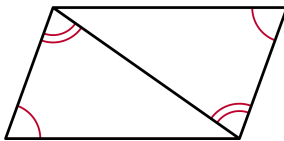


B.

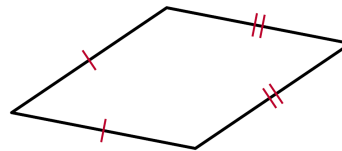


Guided Practice. Determine if the quadrilateral must be a parallelogram. Justify your answer.

7.



8.



Helpful Hint

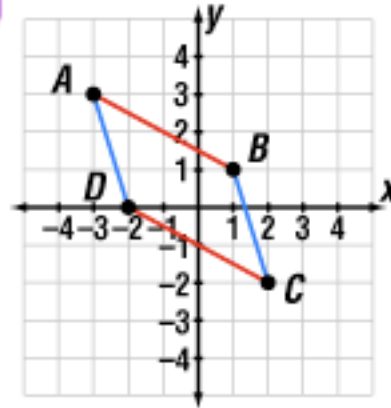
To say that a quadrilateral is a parallelogram *by definition*, you must show that both pairs of opposite sides are parallel.

Refer to video example 3.

Show that the given quadrilateral is a parallelogram by using the given definition or theorem.

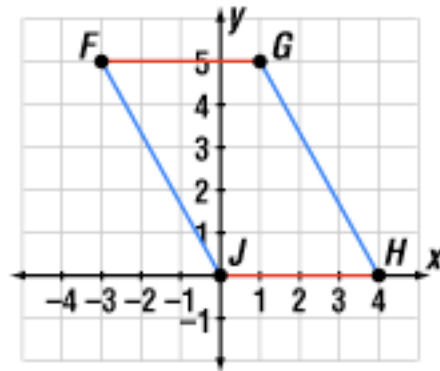
A.

$A(-3, 3)$, $B(1, 1)$, $C(2, -2)$, $D(-2, 0)$



B.

$F(-3, 5)$, $G(2, 5)$, $H(4, 0)$, $J(0, 0)$



3

Proving Parallelograms in the Coordinate Plane

Show that quadrilateral $ABCD$ is a parallelogram by using the given definition or theorem.

A $A(-3, 2)$, $B(-2, 7)$, $C(2, 4)$, $D(1, -1)$; definition of parallelogram

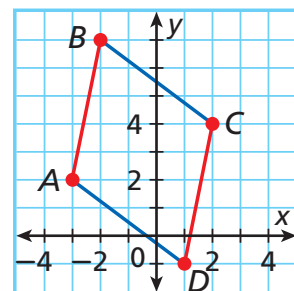
Find the slopes of both pairs of opposite sides.

$$\text{slope of } \overline{AB} = \frac{7 - 2}{-2 - (-3)} = \frac{5}{1} = 5$$

$$\text{slope of } \overline{CD} = \frac{-1 - 4}{1 - 2} = \frac{-5}{-1} = 5$$

$$\text{slope of } \overline{BC} = \frac{4 - 7}{2 - (-2)} = \frac{-3}{4} = -\frac{3}{4}$$

$$\text{slope of } \overline{DA} = \frac{2 - (-1)}{-3 - 1} = \frac{3}{-4} = -\frac{3}{4}$$



Since both pairs of opposite sides are parallel, $ABCD$ is a parallelogram by definition.

B $F(-4, -2)$, $G(-2, 2)$, $H(4, 3)$, $J(2, -1)$; Theorem 6-3-1

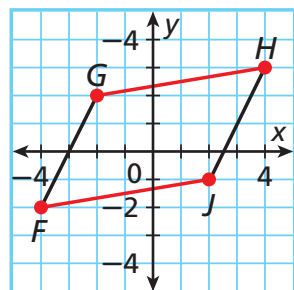
Find the slopes and lengths of one pair of opposite sides.

$$\text{slope of } \overline{GH} = \frac{3 - 2}{4 - (-2)} = \frac{1}{6}$$

$$\text{slope of } \overline{JF} = \frac{-2 - (-1)}{-4 - 2} = \frac{-1}{-6} = \frac{1}{6}$$

$$GH = \sqrt{[4 - (-2)]^2 + (3 - 2)^2} = \sqrt{37}$$

$$JF = \sqrt{(-4 - 2)^2 + [-2 - (-1)]^2} = \sqrt{37}$$



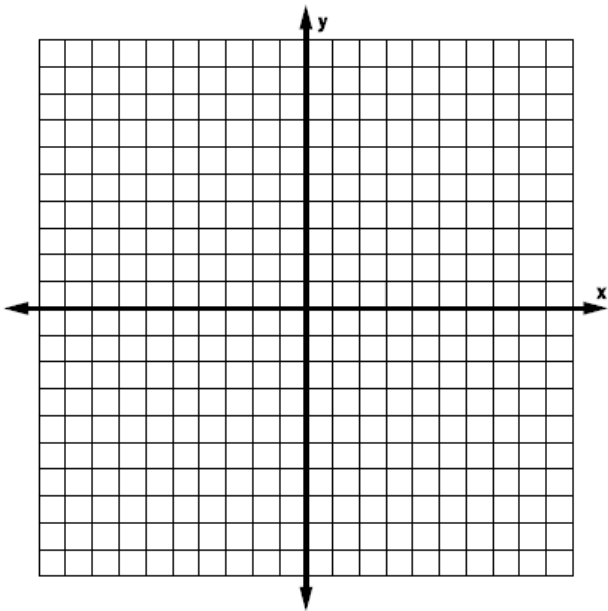
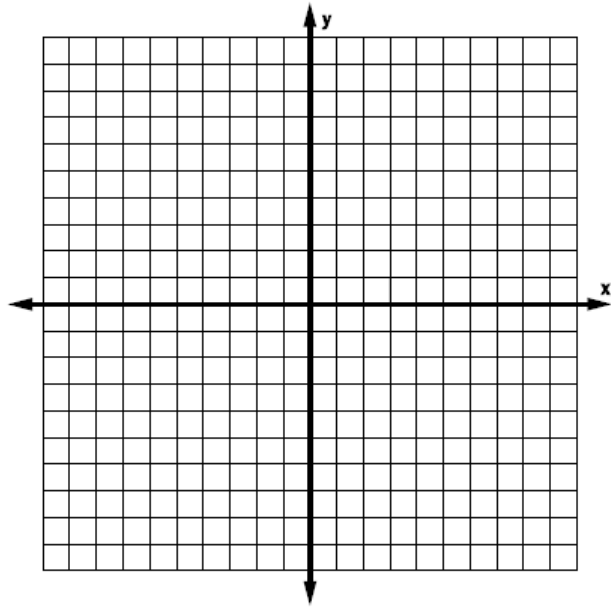
\overline{GH} and \overline{JF} have the same slope, so $\overline{GH} \parallel \overline{JF}$.

Since $GH = JF$, $\overline{GH} \cong \overline{JF}$. So by Theorem 6-3-1, $FGHJ$ is a parallelogram.

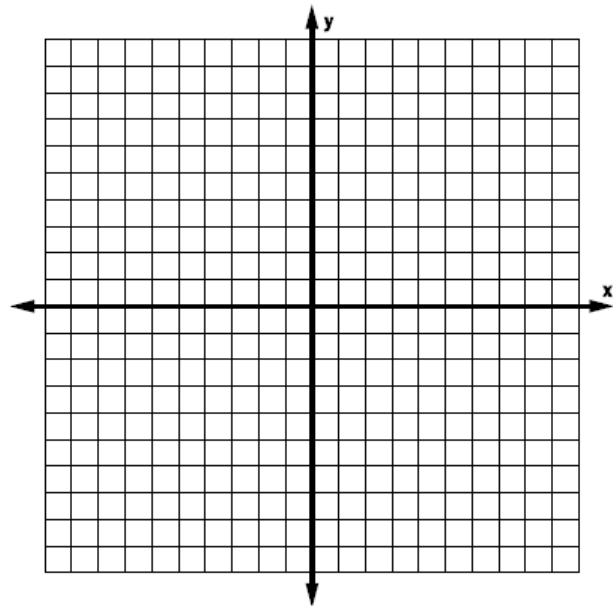
Example 3. Show that quadrilateral JKLM is a parallelogram by using the given definitions or theorems.

A. $J(-1, -6)$, $K(-4, -1)$, $L(4, 5)$, $M(7, 0)$.

B. $A(2, 3)$, $B(6, 2)$, $C(5, 0)$, $D(1, 1)$.



9. Guided Practice. Use the definition of a parallelogram to show that the quadrilateral with vertices $K(-3, 0)$, $L(-5, 7)$, $M(3, 5)$, and $N(5, -2)$ is a parallelogram.



Conditions for Parallelograms

- Both pairs of opposite sides are parallel. (definition)
- One pair of opposite sides are parallel and congruent. (Theorem 6-3-1)
- Both pairs of opposite sides are congruent. (Theorem 6-3-2)
- Both pairs of opposite angles are congruent. (Theorem 6-3-3)
- One angle is supplementary to both of its consecutive angles. (Theorem 6-3-4)
- The diagonals bisect each other. (Theorem 6-3-5)

Helpful Hint

To show that a quadrilateral is a parallelogram, you only have to show that it satisfies one of these sets of conditions.

6-3 Conditions for Parallelograms

- (*p 414*) 9, 10, 11, 13-16, 21-23, 24-25, 34.
- 5A Ready Go Pretest & post-tests.

Question: Did you hear about the quadrilateral who wanted to be a parallelogram?

Answer: Yes, his opposite angles weren't quite up to par.